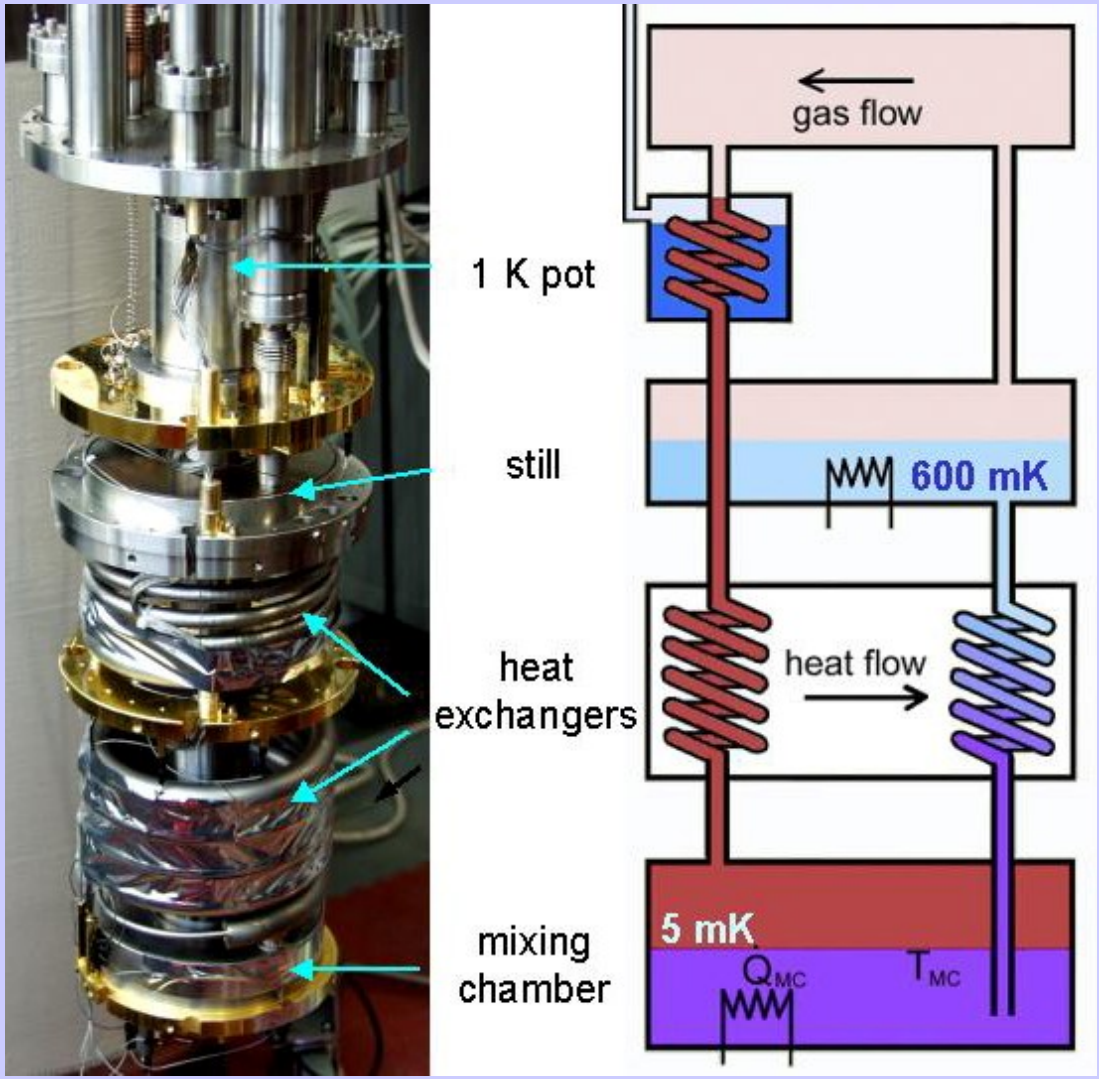
# A numerical analysis of the behavior of sintered heat exchangers

For the IEN-project

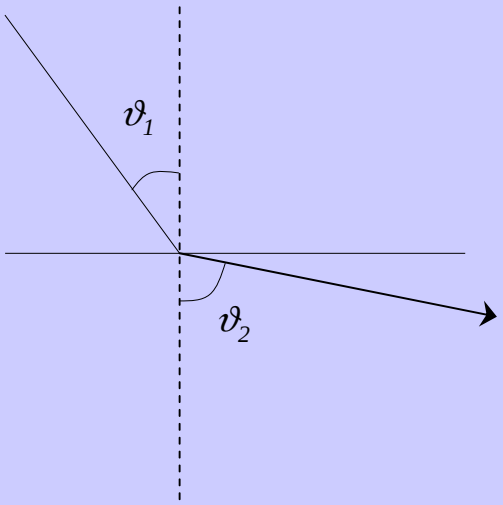
By Bart-Jan Pors


1 K pot
still
heat
exchangers
mixing
chamber
gas flow
600 mK
heat flow
5 mK
$Q_{MC}$
$T_{MC}$

Heat effects in the heat exchanger:

- Heat exchange
- Viscosity
- Thermal conductivity

# Phonon coupling




$$\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{v_1}{v_2}$$

## Heat exchange

Kapitza resistance:

$$R_k = \frac{\alpha_k}{\sigma T^3}$$

$\sigma$ surface area

Heat exchange:

$$\dot{Q} = \frac{\sigma}{4\alpha_k}\left(T_C^4 - T_D^4\right)$$

## Viscosity

$$\Delta \dot{Q}_{visc} = \dot{V} \Delta P$$

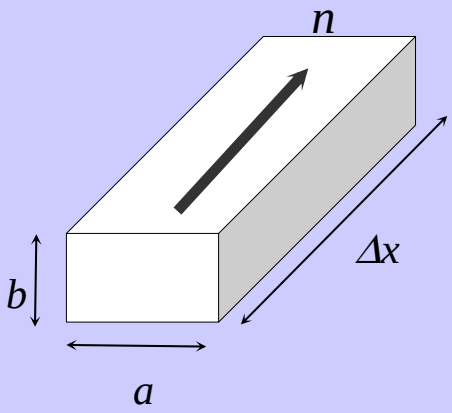
$V$ volume flow, $P$ pressure drop

with $$\Delta P = \Delta Z \eta \dot{V}$$

$\eta$ viscosity, $Z$ geometrical impedance

At $T < 50\ mK$: $$\eta = \eta(T) = \frac{\eta_0}{T^2} [Pa \cdot s]$$

# Geometrical Impedance:




$$\Delta Z = \frac{12}{ab^3}\Delta x$$

## Thermal conductivity

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$

At $T < 50\ mK$: $$k = k(T) = \frac{k_0}{T}[WK^{-1}m^{-1}]$$

# Enthalpy balance

$$\frac{d}{dx}\left(Q_{cond} + Q_{visc} + Q_{exch}\right) = \frac{d}{dx}\left(nH(T)\right)$$

with $dH = C_P dT$

$H$ enthalpy, $C$ specific heat, $\dot{n}$ molar flow

## Frossati's method

$$\frac{d}{dx}\left(Q_{exch}+Q_{visc}+Q_{cond}\right) \rightarrow \frac{d}{dx}Q_{exch}$$

perfect heat exchangers

$$\lambda\frac{d\sigma}{dx}\left(T_C^4-T_D^4\right)=nC_C\frac{dT_C}{dx}$$

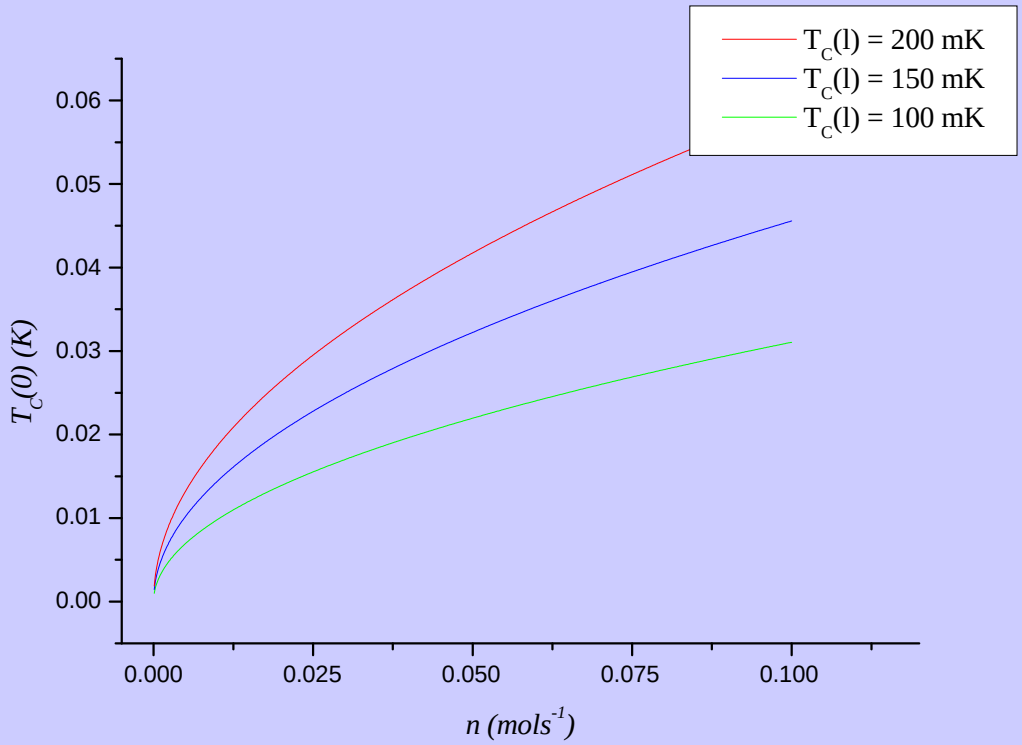
$$dH_C = dH_D$$

## Frossati's method

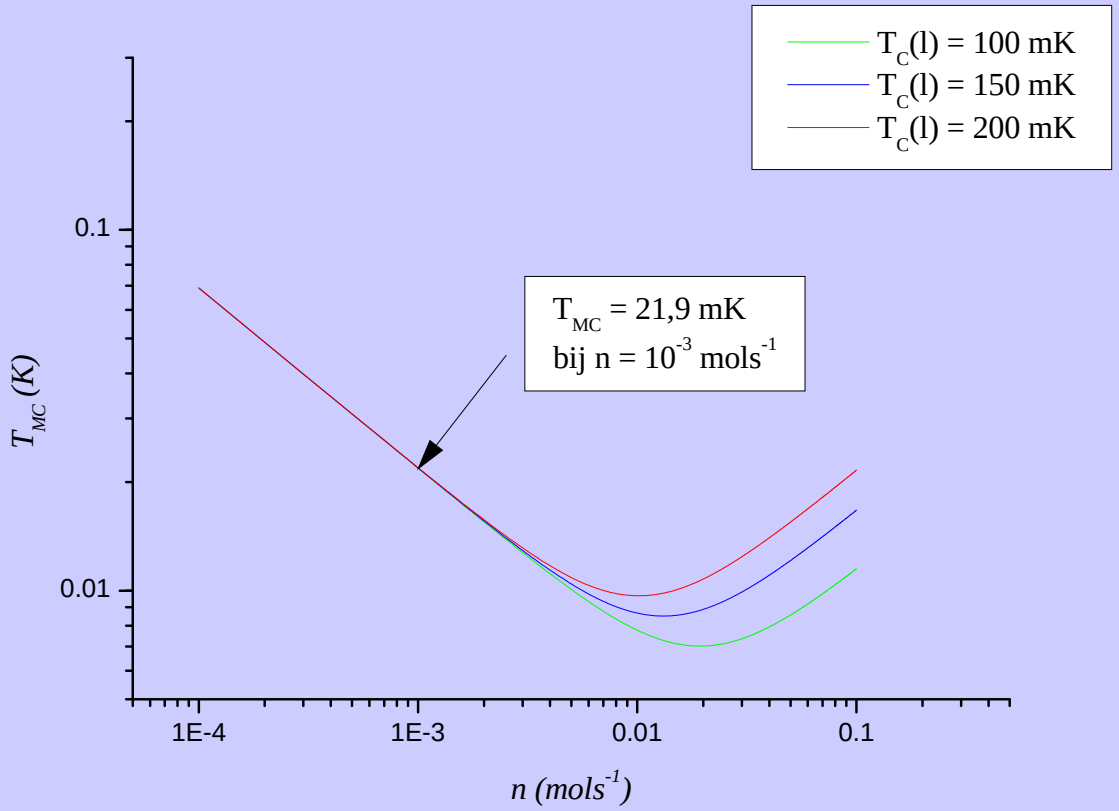
$$T_C^{-2}(0) = \left[ \frac{2\lambda\sigma}{a}\left(1 - \left(\frac{a}{b}\right)^2\right) + T_C^{-2}(l) \right] \frac{1}{\dot{n}}$$

$$\frac{\dot{Q}}{\dot{n}} = 94{,}5\,T_{MC}^2 - 12{,}5\,T_C^2(0)$$

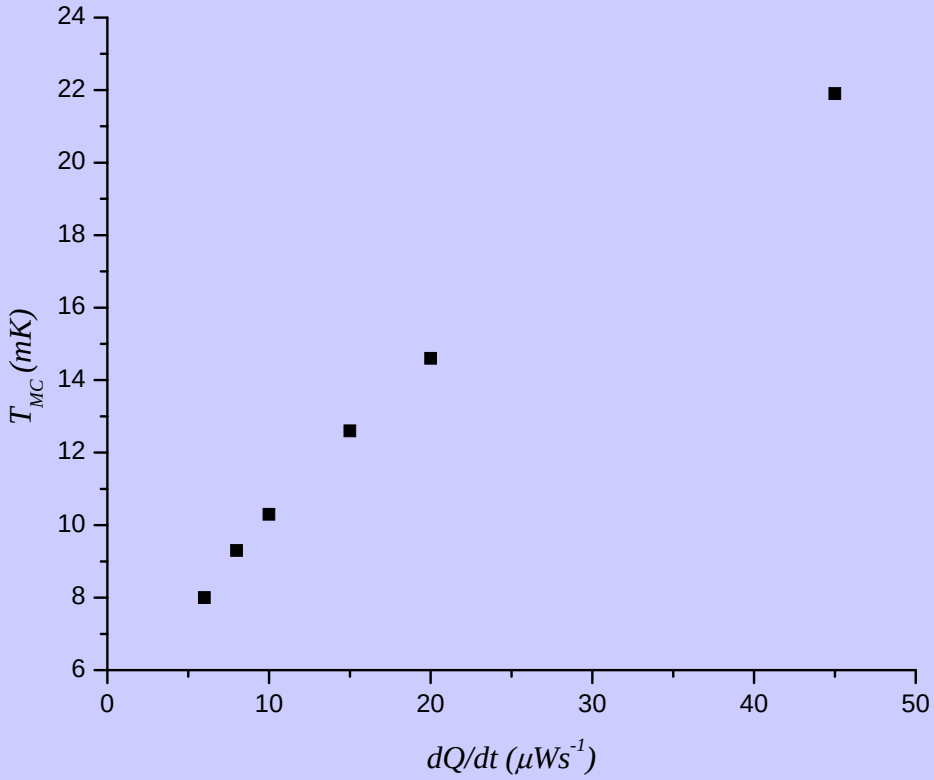
# Frossati's method



# Frossati's method

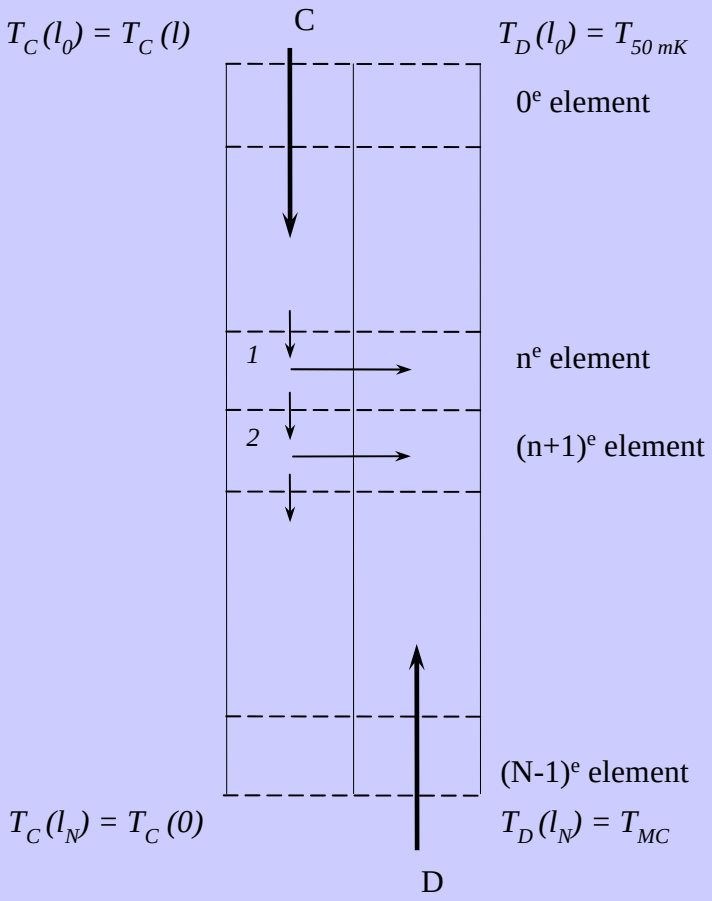


# Frossati's method

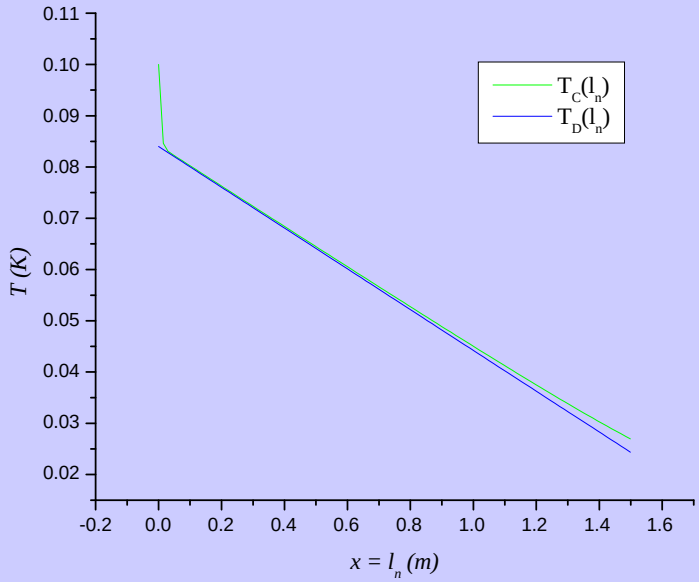



$\sigma \sim 200 - 300\ m^2$, $\dot{n} \sim 10^{-3}\ mols^{-1}$

# Method of imposed temperature profile

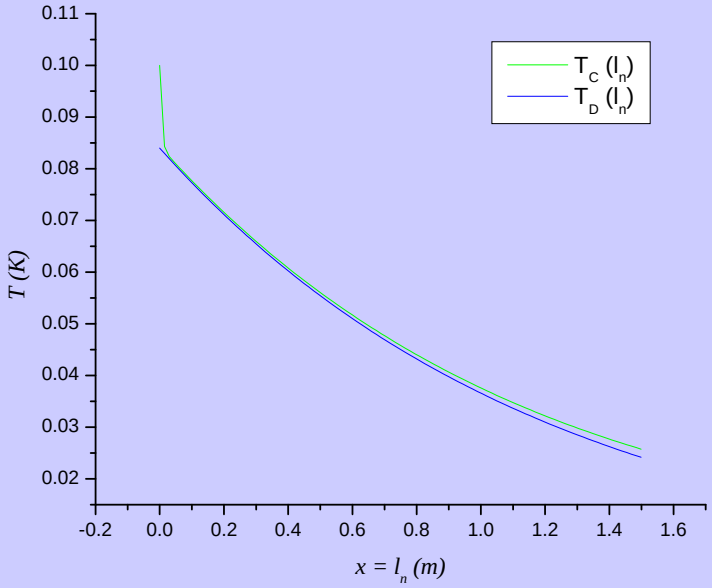


# Method of imposed temperature profile




$$T_C(l_N) = 26{,}96mK$$

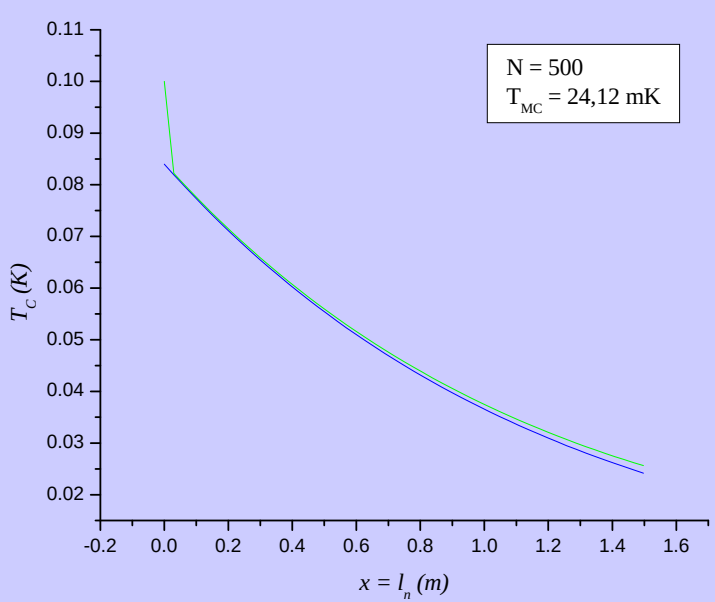
$$T_{MC} = 24{,}31mK$$

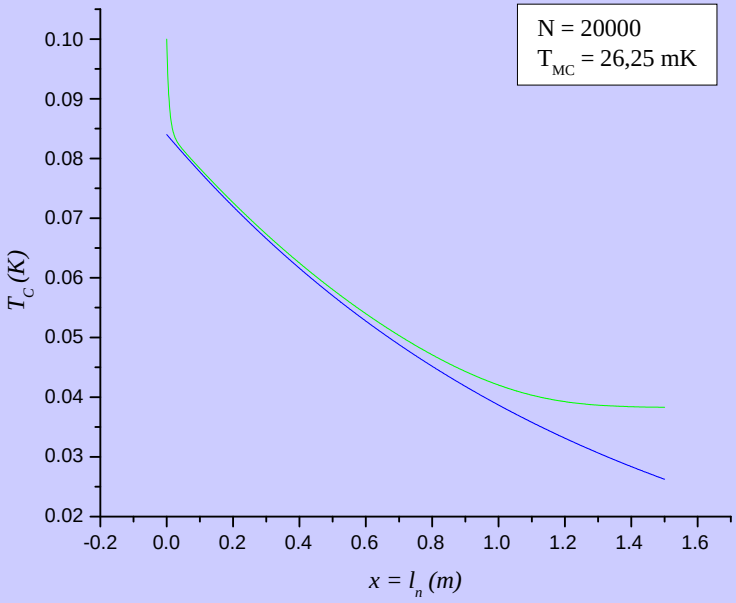



$$T_C(l_N) = 25{,}76mK$$

$$T_{MC} = 24{,}14mK$$

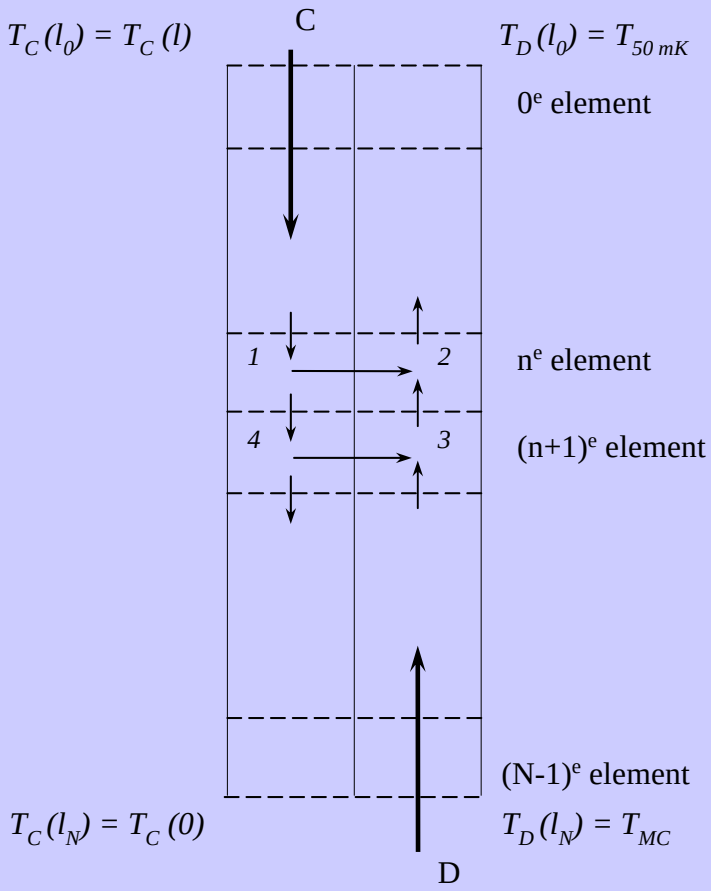
# Method of imposed temperature profile



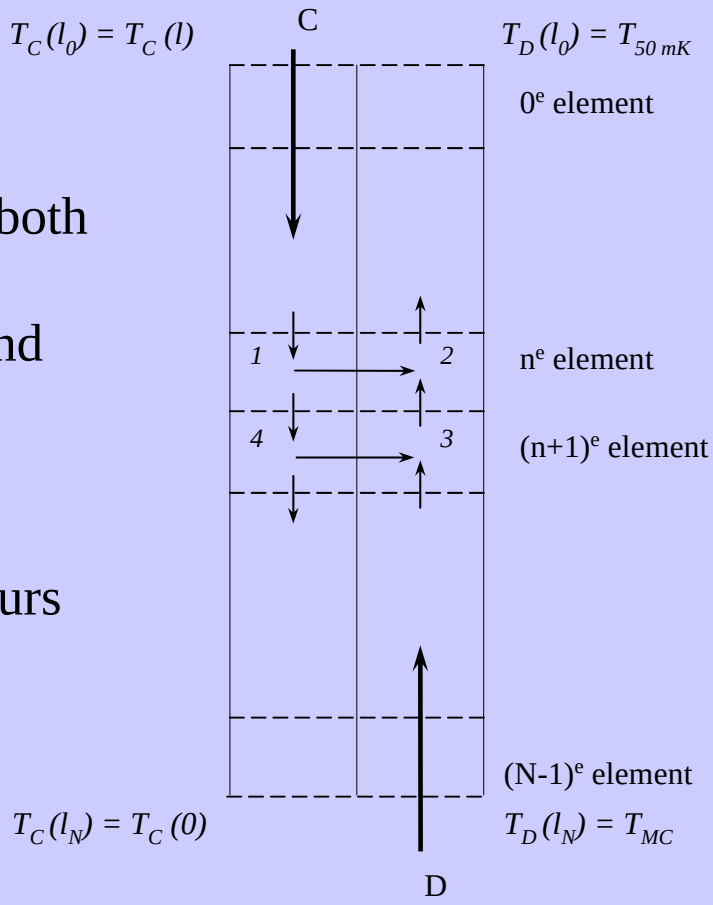

Building up of errors for increasing number of elements

# Method of relaxation to thermal equilibrium



# Method of relaxation to thermal equilibrium

- Impose reasonable temperature profiles on both phases
- Start at $T_C(l_0)$ and calculate all new $T_C(l_n)$ and $T_D(l_n)$
- Compare new and old profiles
- Move values to next elements
- Repeat this procedure until equilibrium occurs



# Accuracy and approximations

*Physical*

- Geometrical impedance for square tube used
- Radial temperature relation (Reynolds)
- Molar volume and heat capacity related to concentration $^{3}He$ in $^{4}He$

*Computational*

- Numerical integration techniques
- Building up errors

*Conceptual*

- Imposed temperature profiles
- Moving values to next elements