MiniGRAIL, A 65 cm Spherical Antenna

Arlette de Waard & Giorgio Frossati

Kamerlingh Onnes Laboratory, Leiden University, Leiden
The Netherlands

Abstract. We intend to build a 65 cm diameter spherical, cryogenic gravitational wave antenna made of the CuAl(6%) alloy with a mass of 1168 Kg, a resonance frequency of 3.7kHz and a bandwidth around 300 Hz. The quantum-limited strain sensitivity $\delta L/L$ would be 2.2×10^{-21} at 10 mK. We believe that a sensitivity around 5×10^{-20} could be reached within the duration of the project, which is (at that frequency) comparable to that of the large interferometers LIGO and VIRGO presently being built. The sources we are aiming at are for instance non-axisymmetric dynamical instabilities of rotating neutron stars within our galaxy, where 10^8 neutron stars are expected to exist.

INTRODUCTION

The advantages of using a spherical detector are considerable and have already led to a number of projects, like GRAIL, the SFERA proposal (Rome), TIGA (LSU) and GRAVITON (Brazil). A sphere has 5 fundamental spheroidal quadrupole modes giving it omnidirectionality and equal sensitivity to both polarizations of the gravitational wave. It can detect the direction and the tensorial character of the wave and it has a very large cross section compared to bar detectors at the same frequency (1),(2),(3),(4).

THE MINIGRAIL

Facilities

The Minigrail facility will be located in the Kamerlingh Onnes Laboratory of the University of Leiden. In our experimental hall five concrete supports are built on vibration-free islands (figure1, left), which stand on pillars, planted 18 meters into the ground and are well isolated from the rest of the building. All the pumps are installed behind the wall on separate concrete blocks, all the pumping lines go through the concrete block on top of the fifth pillar. The Minigrail support (figure1, right) has a hydraulic system installed inside of the four concrete pillars, which carry the 3 tons concrete plate. The hydraulic system can lift the concrete plate with the sphere and suspension 1.5 meters to allow mounting the lower part of the dewar. This system is able to lift 20 tons. The dewar will be supported by a flange attached to the concrete plate.

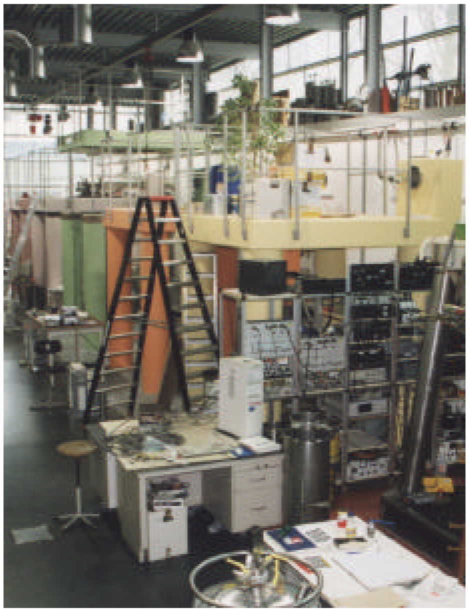

FIGURE 1. Left: Picture of the experimental hall of the Kamerlingh Onnes Laboratry. All the concrete supports are build on 'vibration-free' islands. Right: Picture of the concrete support for the MiniGRAIL. Inside of the pillars a hydraulic system is installed to lift the concrete plate, sphere and suspension. The system is able to lift 20 tons.

Design of the MiniGRAIL Antenna

A schematic picture of the MiniGRAIL detector is shown in figure 2. The sphere will be made of CuAl(6%) because of its high quality factor of 15 million and good thermal conductivity at low temperatures since it doesn't become superconducting. This material can be cast in the Netherlands by the company LIPS in Drunen. The antenna will have a diameter of 65 cm and a weight of 1168 kg. It will be suspended from the center with a copper rod with a diameter of 10 mm. The vibration isolation stages consist of seven masses, the upper four made of CuAl(6%) and the last three made of copper. The masses will be connected to each other by springs or rods. Each stage will have a mass of about 120 kg and will give an attenuation of about 40 dB, so the total attenuation will be in the order of 300 dB at the resonant frequency of the sphere. The total system will be suspended by three titanium rods attached to the flange of the dewar. The dilution refrigerator (dr) will be off center, see figure 2, and the mixing chamber will be thermally anchored by copper 'spaghetti' to mass number 5, the first copper mass, of the vibration isolation system, so vibrations coming from the dr will be attenuated by 120 dB. The heat exchanger of the dr will be a large circular tube to avoid instabilities coming from convection of the ^{3}He inside of the dilute phase of the mixture. The dewar will be specially built for this purpose by Kadel (5). The He consumption will be less than 30 liter/day. The volume of the He reservoir is 300 liters, so He transfer will be needed every 10 days. The feed through of the 1K pot pumping line, the He and N_2 transfer lines are not shown in the picture.

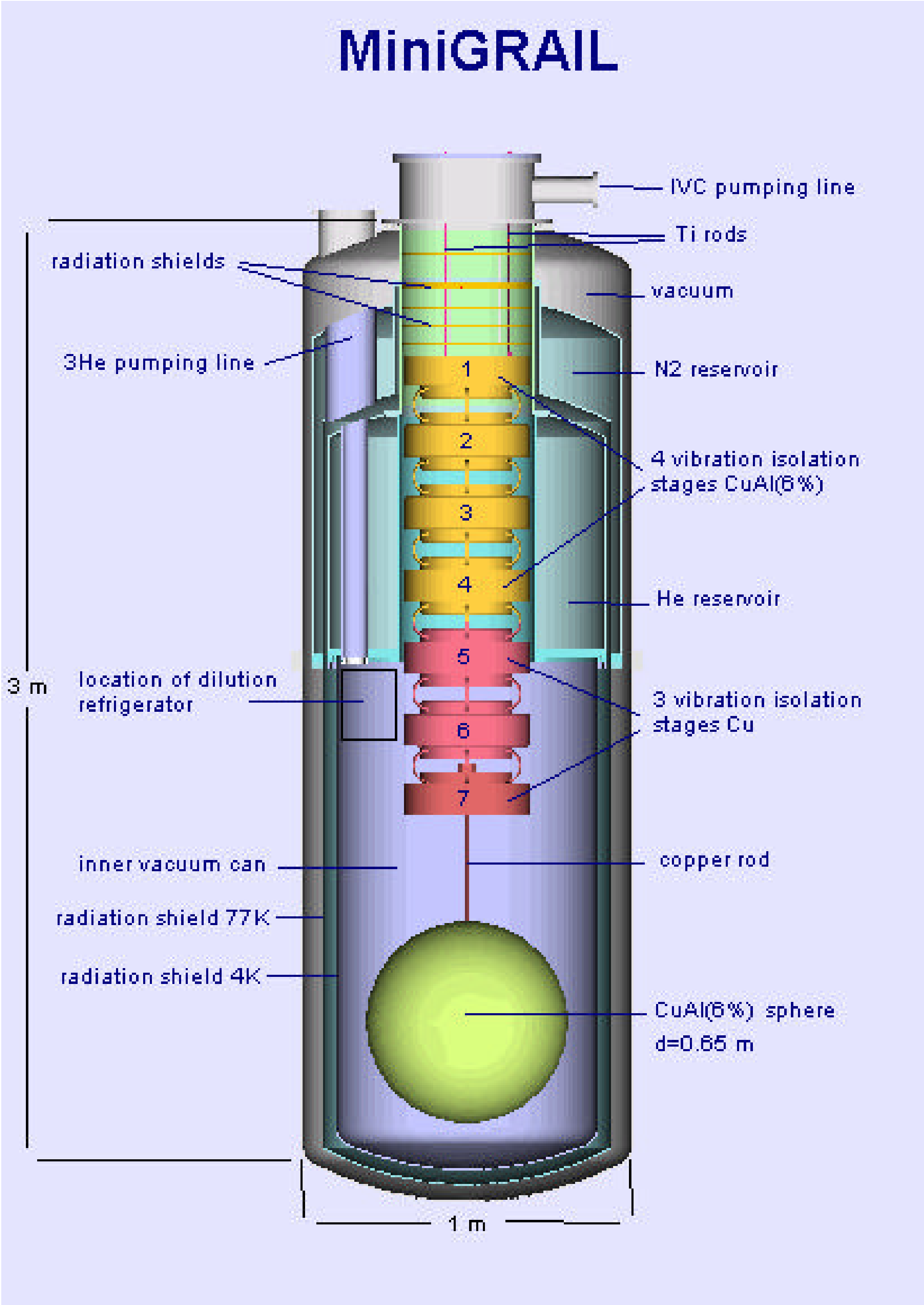

FIGURE 2. Schematic layout of the dewar with the antenna and the seven stages suspension. The upper four stages are made of CuAl(6%). The third copper mass will be thermally anchored to the mixing chamber of the dilution refrigerator. The dewar will be specially made by Kadel engineering.

MiniGRAIL sensitivity

The quadrupolar frequencies of a sphere are given by (4),(6)

$$w_n = \frac{c_n v_s}{R_s} \tag{1}$$

where n is the order of the quadrupole mode, the numerical coefficients c_1=1.62 and c_2=3.12, v_s= 4620m/s the sound velocity of CuAl(6%) and the radius of the sphere R_s =0.325m, so the first quadrupolar frequency is $f_1 \cong$ 3665 Hz.

The gravitational wave energy absorbed by a resonant antenna is expressed in terms of its cross section. For a spherical antenna, which has 5 fundamental spheroidal quadrupole modes the cross section is given by:

$$s_n = F_n \frac{G}{c^3} M_s v_s^{\;2} \tag{2}$$

F_1=2.98 and F_2=1.14 are dimensionless coefficients for the first and second quadrupole mode, G=6.7x10^{-11} m^3/kgs is the gravitational constant and c=3.0x10^8 m/s is the speed of light, M_s=1168 kg is the mass of the sphere. The cross section for the first quadrupolar mode is $\sigma_1 \cong 1.04 \times 10^{-25}$ m^2/s. The initial goal would be to cool the sphere down to 20 mK and using a SQUID with an energy sensitivity of E_N=100hν so that the noise temperature of the amplifier would be

$$T_N = \frac{E_N}{k_B} \cong 1.76x10^{-5} K \tag{3}$$

The minimal energy a resonant antenna is able to detect, is given by in terms of the effective temperature, by (4)

$$T_{eff} \cong 2\sqrt{2} T_N \left(1 + \frac{2T}{bQT_N}\right)^{\frac{1}{2}} \approx 4.99x10^{-5} K \tag{4}$$

with T≈20 mK, the thermodynamic temperature, β≈0.05 is the ratio of the energy absorbed by the antenna and that transferred to the transducer and Q≈1.5x10^7 (7) is the mechanical quality factor of CuAl(6%) at 20 mK.

We consider the sensitivity to a gravitational wave burst of a duration τ_g. The Fourier transform H(f) of the burst is assumed constant within a bandwidth Δf so

$$H(f) \cong H(f_0) = H_0 \cong \tfrac{1}{2} h_0 \tau_g$$

For a SNR=1 we have

$$H_0^{\min} = \left[\frac{kT_{eff}}{\frac{1}{2} m l^2 (2pf_1)^4}\right]^{\frac{1}{2}} \cong 1.05x10^{-23} \tag{5}$$

where m=M_s=1168 kg is the mass of the detector, l=0.6R=0.195m, f_1 is the resonant frequency of the first quadrupolar mode. The strain sensitivity to a gravitational wave burst of duration τ_g =1.5 ms is

$$h_0^{\min} \cong \frac{2H_0}{t_g} \approx 1.4x10^{-20} \qquad (6)$$

The bandwidth is calculated with

$$\Delta f = 0.7 f_1 b \left(1 + \frac{2T}{bQT_N}\right)^{\frac{1}{2}} \cong 128Hz \qquad (7)$$

and a spectral amplitude of

$$\tilde{h} = \sqrt{S_h(f_1)} = \sqrt{2p\Delta f_1} t_g h_0^{\min} \approx 6.0x10^{-22} \qquad (8)$$

We also calculated the values for the quantum limited SQUID sensitivity of 1hυ at a temperature of 10 mK. In the quantum limit the strain amplitude h_0^{min} =$1.5x10^{-21}$. When we look at the Heisemberg uncertainty principle $\Delta x \Delta p \geq h/2$? $(\Delta x^2) \geq h/2M_S\omega$, so $\Delta x_{QL}=1.4x10^{-21}$ m and the strain amplitude $h=(\Delta x/x)\cong 2.15x10^{-21}$. From this we can see that the strain amplitude of a sphere with ϕ=65 cm is limited by the Heisemberg uncertainty principle, although only by 20%.

In table 1 there is an overview of all the parameters of the Minigrail for the initial goal, reaching a thermodynamic temperature of the antenna of about 20 mK and using a SQUID with an energy sensitivity of 100hυ and of the quantum limited case at a temperature of 10 mK. The mass $M_1=M_{eff}=0.25M$ is the effective mass of the equivalent sphere quadrupole oscillator (8) and M2 and M3 are the two resonant masses of the motion transducer, calculated so that $\Delta F \sim 2F_1(M_3/M_2)^{1/2}$, (9).

TABLE 1. Minigrail parameters.

		Initial goal	**Quantum limit**
Radius	R	0.325 m	
Material		CuAl(6%)	
Sound velocity	v_s	4620 m/s	
Density	ρ	8121 kg/m^3	
Thermod. temperature	T	0.02 K	0.01 K
Quality factor	Q	$1.5x10^7$	
Mass	M	1168 kg	
Effective mass	M_1=0.25M	293 kg	
Mass of transducer 1	M_2	2.3 kg	
Mass of transducer 2	M_3	0.023 kg	
Resonant frequency	F_1	3665 Hz	
	β	0.05	0.3
	$T/\beta Q$	$2.6x10^{-8}$ K	$2.2x10^{-9}$ K
Energy sensitivity	E_N (hν)	100 hν	1 hν
Noise temperature	$T_N = E_N/k$	$1.76x10^{-5}$	$1.76x10^{-7}$
Effective temperature	T_{eff}	$4.99x10^{-5}$ K	$5.3x10^{-7}$ K
	T/QT_N	$0.76x10^{-4}$	$3.8x10^{-3}$
Freq. band width	Δf	128 Hz	779 Hz
Strain amplitude	h_0^{min} (τ=1.5ms)	$1.4x10^{-20}$	$1.4x10^{-21}$
Spectral amplitude	$\sqrt{S(f_0)}$ (Hz)$^{-1/2}$	$6.0x10^{-22}$	$1.5x10^{-22}$

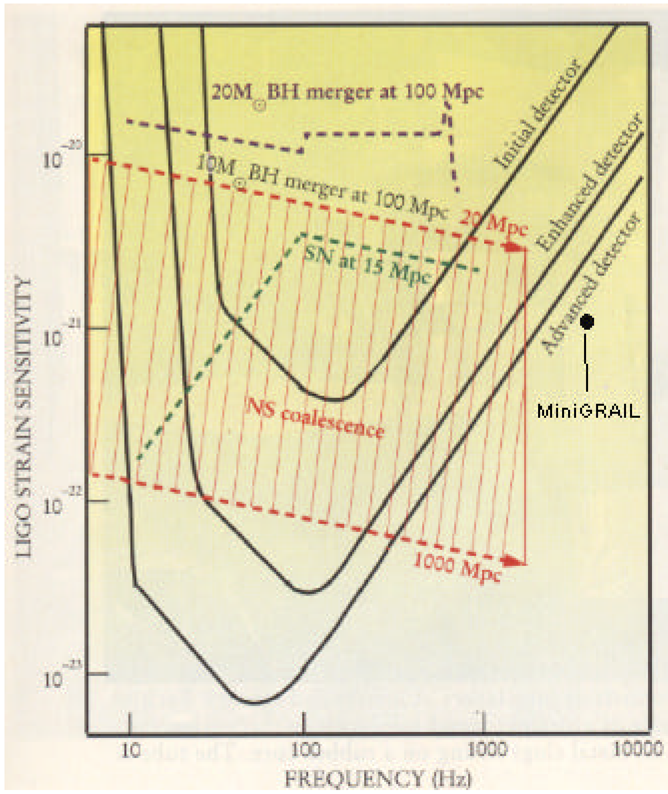

FIGURE 3. Minigrail sensitivity compared to the sensitivity of the advanced LIGO detector (10).

Cryogenics

Cooling from 300 K to 4.2 K

From the Nautilus and Auriga experiments we know that it takes about 20 days to cool down a 2.3 ton Al5056 antenna with a surface area of ~$6m^2$ from room temperature to 77K (11),(12). Because $c_{CuAl} \approx 2c_{Al}$, the total heat capacity of Minigrail is about the same as that of the bars, though the contact area is only $1.1m^2$. Cooling the sphere with exchange gas would take more than 3.5 months (110 days) ,(13). The total enthalpy that has to be removed is in the order of 10^8 J. Using a forced helium flow, which is able to remove $500W/m^2$, the time needed to cool down the sphere to 77 K could be reduced to $t=(10^8/500)=2x10^5 sec \approx 2.5 days$.

The enthalpy of copper at nitrogen temperature is H_{77K}=6J/g, so the total enthalpy of the sphere will be about $7x10^3$J, $t=7x10^6/500=1.4x10^4 sec \approx 4$ hours. The total helium consumption from room temperature to 4.2 K will be around 600 liters.

Cooling from 4.2 K to 10 mK

Taking the specific heat of copper at 4K as $c_{Cu}=1.3\times10^{-5}$ J/cm^3

$$H = \int CdT = 10^{-4}\frac{J}{g} \Rightarrow H \sim 100\,J \qquad (9)$$

The cooling power of a dilution refrigerator will be about 1 mW at 100 mK, so it will take about 30 hours to cool down to 100 mK. The enthalpy at 100 mK is about 0.1J and the cooling power of the dr 10μW at 10 mK and so t<3 hours.
Using a forced helium flow and a powerful dilution refrigerator, the Minigrail can be cooled from room temperature down to 10 mK in 4 days so tests and experiments can be done very quickly and at low costs.

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